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Analysis of α-, β,- and γ-Ray Emission Probabilities

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ANALYSIS OF α-, β-, AND γ-RAY EMISSION PROBABILITIES

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For the analysis of decay-scheme data, it is desirable to utilize all available α-particle, β-particle, and γ-ray data when determining emission probabilities. Additional constraints from the decay scheme should also be considered. In this paper, the inclusion of all data in a fully constrained, covariance analysis to obtain a self-consistent set of emission probabilities will be demonstrated. The analysis of data where the covariance error matrix is unknown will be discussed.

1. Perspective

A common problem with the normalization of decay scheme intensities is obtaining a self-consistent set of α-, β-, and γ-ray branchings. For example, normalizing γ-ray intensities to the total ground-state feeding does not, in general, correctly normalize the β or α intensities. Because of statistical fluctuations, decay branchings through levels may be negative (non-physical) or have positive values that are inconsistent with log*ft* systematics. In some cases, the level scheme is incomplete so statistical analysis is impractical. Browne has demonstrated covariant analysis techniques for normalizing γ-ray intensities [1] and beta feedings [2]. These methods do not take into account non-physical decay branchings resulting from statistical fluctuations. This paper discusses a self-consistent method for constraining the α-, β-, and γ-ray feedings so that all of the normalizations coincide. A re-analysis of the data for ^{133}Ba is presented as an example.

2. Method of calculation

For levels with known α- or β- feedings, the intensity balance can be constrained to zero. The equation describing the intensity balance through the ith level can be written as

$$\sum_{j=1}^{i-1} I_{ij}(\gamma+ce) = \sum_{k=i+1}^{n-1} I_{ki}(\gamma+ce) + I_{ni}(\alpha,\beta) \tag{1}$$

where i,j,k are level sequence numbers numbering from 1 for the ground-state to n for the highest level (isomer, α or β parent) considered in the analysis. $I_{ij}(\gamma+e)$ are the electromagnetic transition intensities and $I_{ni}(\alpha,\beta)$ are the α- or β- intensities feeding the levels. A consequence of the constraint of eq. (1) is that the intensities feeding and deexciting the level are over-specified. To illustrate this, consider the decay scheme for ^{133}Ba [3] shown in fig. 1. The ground state and first two excited states (levels 1-3) receive no β-feeding because of the highly forbidden nature of those decays. For the 160.6-keV level (level 3), the constraint

$$I_{31}(\gamma+e) = I_{43}(\gamma+e) + I_{53}(\gamma+e) - I_{32}(\gamma+e) \tag{2}$$

can be applied. A similar constraint can be applied to the 81.0-keV level (level 2). The transition intensity parameters I_i are linearly related to the experimental values $\hat{I}_i$ by a set of equations which can be written in matrix notation as

$$\begin{bmatrix} \hat{I}_{21} \\ \hat{I}_{32} \\ \hat{I}_{31} \\ \hat{I}_{43} \\ \hat{I}_{42} \\ \hat{I}_{41} \\ \hat{I}_{54} \\ \hat{I}_{53} \\ \hat{I}_{52} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix}. \tag{3}$$

Here, the 81.0- and 160.6-keV levels are constrained to zero net feeding so that seven parameters (γ-ray intensities) are sufficient to describe the nine transitions. The constraints are analogous to those imposed when γ-ray energies are fit to a level scheme [4]. If the α- or β- intensities are known on the same relative intensity scale as the electromagnetic transitions, the constraint can be applied to all levels. The choice of which transitions are parameters is arbitrary, depending on how we solve the linear equations. In eq. (3), the seven parameters correspond to the intensities of the second and fourth through ninth transitions. The

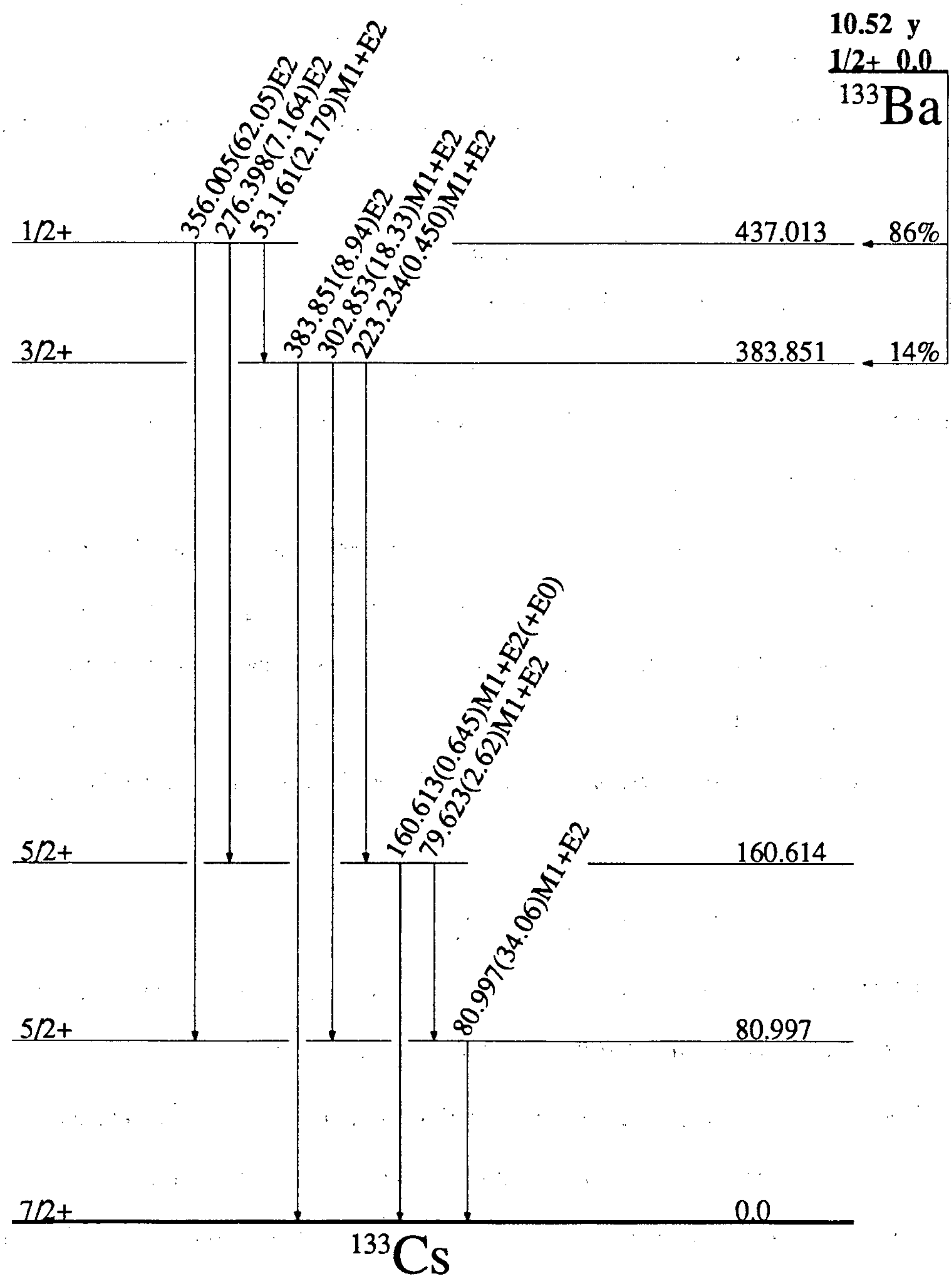

Fig. 1. Decay scheme for ^{133}Ba from Nucl. Data Sheets 49, (1986) 639.

matrix element λ_{ij} is +1 for parameters which feed the jth level, −1 for parameters deexciting the ith level, and 0 if the parameter does not involve the level. If we solve the linear relations in eq. (3) to obtain the parameters, a new set of fitted γ-ray intensities $\tilde{I}_i$ can be determined from the equation

$$\tilde{I}_i=\sum_{j=1}^{m}\lambda_{ij}I_j \tag{4}$$

where m is the number of intensity parameters defined.

For a general decay scheme, additional constraints and subsequently fewer parameters I_j may arise. Eq. (4) can be written in matrix notation as

$$\hat{I}=\lambda I \tag{5}$$

The solution to eq. (5) is a well known linear regression problem where

$$I=(\lambda^T\lambda)^{-1}\lambda^T\hat{I} \tag{6}$$

Eq. (6) can be weighted by substituting $\lambda^T W$ for λ^T where W is the weighting matrix which will be discussed in greater detail below.

The above solution determines the parameters I_i which correspond to some of the intensities. These parameters are analogous to the level energies that are determined by fitting the γ-ray energies to the level scheme. To determine all of the intensities, the calculated parameters should be substituted into eq. (4) and new $\tilde{I}_i$ values calculated. If a weighted analysis is performed, the covariant error matrix $V=(\lambda^T W\lambda)^{-1}$ contains the covariance relationships necessary to calculate the uncertainties in the fitted intensities. These uncertainties are

$$\Delta\tilde{I}_i^2=\sum_{k}^{n}\lambda_{ik}^2 V_{kk}+\sum_{k}^{n}\sum_{l\neq k}^{n}\lambda_{ik}\lambda_{il}V_{kl} \tag{7}$$

where the first term to the right of eq. (7) contains the variances of the parameters that define the transition intensity, and the second term contains the covariance between those parameters.

Additional α- or β-branchings, not entered explicitly in the above analysis, can be calculated from the intensity parameters and the covariant error matrix. The relative branching intensity to any level is simply the sum of transition intensities deexciting the level minus the sum of intensities feeding that level. Summing over the intensities defined by eq. (4), we obtain

$$\bar{I}_i(\alpha,\beta)=\sum_i^l\sum_j^m\kappa_j\lambda_{ij}I_j=\sum_j^m\beta_{ij}I_j \tag{8}$$

where the summation is over all γ rays populating ($\kappa_j=1$) or deexciting ($\kappa_j=-1$) each level. The vector β defining the α- or β-transition intensity is analogous to the λ vectors that form the rows in the λ matrix and define the transition intensities. The elements of β_{ij} are simply the sum of the corresponding λ_{ij}'s associated with the level. The uncertainty in the branchings is determined from the covariant error matrix, in analogy with eq. (7), and is given by

$$\Delta\bar{I}_i^2(\alpha,\beta)=\sum_k^n\beta_{ik}^2V_{kk}+\sum_k^n\sum_{l,l\neq k}^n\beta_{ik}\beta_{il}V_{kl} \tag{9}$$

The statistical analysis discussed above provides a self-consistent set of relative intensities on an arbitrary scale. In order to determine the absolute branching intensities per decay, it is necessary to renormalize them. This normalization is an additional constraint and the normalization constant N can be determined from a sum of transition intensities known to carry 100% of the decay intensity. Commonly, this is satisfied by transitions feeding the ground-state(s) (plus sometimes long-lived isomeric states) of the daughter(s). We define the normalization constant N as

$$N=\frac{1}{\sum_i^n\sum_j^m\lambda_{ij}I_j} \tag{10}$$

where the initial summation is over all transitions contributing to the normalization. If the normalization includes only ground-state transitions, eq. (10) becomes

$$N=\frac{-1}{\bar{I}_1(\alpha,\beta)} \tag{11}$$

The negative sign arises because the apparent β or α feeding to the ground state is negative as defined in eq. (8). The uncertainty ΔN in the normalization is determined by eq. (9), and absolute transition intensities are given by

$$\bar{I}_i=N\tilde{I}_i \tag{12}$$

The uncertainty in the absolute transition intensities $\bar{I}_i$ are defined as

$$\Delta\bar{I}_i^2=\bar{I}_i^2\left[(\frac{\Delta N}{N})^2+(\frac{\Delta\tilde{I}_i}{\tilde{I}_i})^2+\frac{2cov(N,\tilde{I}_i)}{N\tilde{I}_i}\right] \tag{13}$$

where the covariant terms in eq. (13) are

$$cov(N, \bar{I}_i) = \sum_{j}^{m} \sum_{k=1}^{m} \lambda_{ij} \beta_{1k} V_{jk} \tag{14}$$

Similarly, the absolute photon branching intensities are determined from the absolute intensities and the experimental conversion coefficients by

$$\bar{I}_i(\gamma) = \frac{\bar{I}_i}{(1+\alpha_i)} \tag{15}$$

The uncertainty in the photon branching intensity can be determined in analogy with eq. (13) as

$$\Delta\bar{I}_i^2(\gamma) = \bar{I}_i^2(\gamma) \left[\left(\frac{\Delta N}{N}\right)^2 + \left(\frac{\Delta\hat{I}_i(\gamma)}{\hat{I}_i(\gamma)}\right)^2 + \frac{2cov(N, \bar{I}_i)}{N\bar{I}_i} \left(\frac{\Delta\hat{I}_i(\gamma)}{\Delta\bar{I}_i}\right)^2 \right] \tag{16}$$

where we have assumed that the experimental uncertainties for the γ rays and conversion coefficients are independent and were added in quadrature to determine the weighting matrix. The absolute transition probabilities described above yield identical results to those described by Browne [1,2] when no constraints are applied to the level scheme.

3. Calculation considerations

Although the mathematical considerations discussed in sec. 2 are generally valid, the application of these methods to real data requires additional considerations which are outlined below.

(1) Completeness of decay schemes

The calculations described here presume that the variations in the branching intensities are statistical. If unobserved transitions populate the constrained levels, the intensity adjustments will be invalid. This problem will often occur in decays of nuclei far from stability where Q-values are large and, as a result, numerous, weak transitions may have a large cumulative effect. Even near stability, if the level density in the daughter is high or the transition energies occur where the intensity is difficult to measure, potential pitfalls exist. In addition, an accurate knowledge of conversion coefficients and the existence of E0 transitions is required to correctly fit the data.

(2) The weighting matrix

The calculations described here presume that the full weighting matrix relating the input data is known. If the transition intensities were uncorrelated, a diagonal weighting matrix, containing the weights for each input intensity, would be sufficient. In general, the measured intensities should be correlated, first to the experimental efficiency calibrations, and ultimately to the standard source calibrations. If we know these relationships, the off-diagonal weighting matrix elements can be calculated and included in the analysis. Unfortunately, this is only possible when the appropriate information is gathered by the experimenter. Analysis of published data cannot, generally, include a sophisticated analysis of these correlations. For the example presented in this paper, we have assumed a diagonal weighting matrix where the experimental values are weighed by their uncertainties. This choice of weighting may not always lead to valid results. For weak transitions, the assumption of a gaussian probability distribution allows the solution to yield non-physical, negative intensities.

(3) Model testing

In light of the caveats expressed above, it is useful to test the data for the quality of the fit. Many methods of testing statistical data have been developed. One method, which was used for the *Table of Radioactive Isotopes* [5], is a simple chi-square analysis. In this analysis we calculate the quantity χ^2/f where

$$\frac{\chi^2}{f}=\frac{1}{m}\sum_{i=1}^{n}\frac{(\hat{I}_i-\tilde{I}_i)^2}{\Delta\hat{I}_i^2} \tag{17}$$

and $\hat{I}$,$\Delta\hat{I}$ are the experimental intensity and its uncertainty, $\tilde{I}$ is the fitted intensity, n is the number if transitions, and m is the number of fitted parameters. This test can indicate whether the fitting assumptions agree with the data within experimental accuracy. Typically, for $\chi^2/f<1$, a >95% confidence limit can be assumed for the result. Additional tests for correlations in the data can be performed when sufficient data exist.

(4) Measurements on differing scales

We alluded in sec. 2 to the possibility of constraining the level branchings to the measured α- or β-feeding intensities. If those measurements are on the same relative intensity scale as the electromagnetic transitions, the discussion in sec. 2 is adequate. Otherwise, the intensities must be placed on a common scale. A procedure for doing this has been outlined by Tepel [6]. The goal is to renormalize the data such that the normalization minimizes χ^2/f. This is a non-linear process where an approximate normalization constant is chosen, χ^2/f is calculated, and the constant is then iterated until a minimum value of χ^2/f is found. Work is currently in progress to implement this capability into GAMBET [7], the computer code written for these calculations, and will be reported at a later date.

4. ^{133}Ba Example

The experimental data from ref. [8] is summarized in table 1. The computer code GAMBET [7] has been used to perform the analysis described above for ^{133}Ba decay. The resulting γ-ray intensities from an unconstrained analysis are given col. 4 of table 1. These values are identical to those calculated by the method of ref. [1], but they have much larger uncertainties than in ref. 8. The larger, unconstrained uncertainties result mainly from the conversion coefficient uncertainties. It should be noted that the uncertainties in ref. [8] were obtained by averaging many values with >2% individual uncertainties. If significant correlations exist between the measurements, e.g. due to common calibration standards, simple averaging of the values will artificially remove the covariant errors.

The last column in table 1 contains the absolute γ-ray branching intensities, constraining the β-feeding to the 81.0- and 160.6-keV levels to zero, and normalizing to the ground-state transition intensity to 100.0%.

Table 1. Comparison of ^{133}Ba γ-ray emission probabilities.

			Photon Branching Intensity		
E(level)	E_γ	α(tot)	Ref. [8]	Unconstrained	Constrained
81.0	81.0	1.63(6)	34.06(27)	34.22(91)	34.23(28)
160.6	79.6	1.70(6)	2.62(6)	2.63(9)	2.68(6)
	160.6	0.296(3)	0.645(8)	0.648(20)	0.646(8)
383.9	223.2	0.0984(1)	0.450(4)	0.452(13)	0.450(4)
	302.9	0.0438	18.33(6)	18.42(51)	18.34(7)
	383.9	0.0203	8.94(3)	8.98(25)	8.94(3)
437.0	53.2	6.0(3)	2.179(22)	2.189(64)	2.180(22)
	276.4	0.0569	7.164(22)	7.20(20)	7.166(27)
	356.0	0.0255	62.05(19)	62.3(17)	62.08(23)

The statistical uncertainties of the constrained values are reduced to nearly the magnitude of the values in ref. 8. These uncertainties may contain systematic uncertainties from sources such as detector calibrations and summing. For the constrained analysis, χ^2/f =0.23 which yields a 98% confidence limit justifying the analysis in this case.

In table 2, the β-branchings for ^{133}Ba decay are presented. The first column indicates the evaluated β-feedings from Nuclear Data Sheets [3]. The source of the uncertainties was not indicated in ref. 3, however it is apparent that the limits on feedings to the 81.0- and 160.6-keV levels are experimental. Log*ft* selection rules [9] suggest that a log*ft*>12.8 is expected, limiting these feedings to less than 0.003%. The second column indicates the β-feeding derived assuming an unconstrained fit. Here, non-physical negative feeding is derived to the 81.0- and 160.6-keV levels and the total β-feeding to the other levels is 100.4%. The last column indicates the constrained β-feedings. The ground state and first two excited states receive no feeding and the β-feedings to the higher states sum exactly to 100.0%. The result of the constrained analysis agrees closely with that of ref. [2] where a partial constraint was applied.

Table 2. Comparison of ^{133}Ba β-emission probabilities

	β-feeding		
E(level)	Ref. 3	Unconstrained	Constrained
0.0	0	0	0
81.0	<0.3	-0.26(20)	0
160.6	<3	-0.1567(44)	0
383.9	14(1)	13.55(87)	13.50(78)
437.0	86(1)	86.9(25)	86.50(78)

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